ASYMPTOTICALLY OPTIMAL APPROACH FOR THE MAXIMUM SPANNING TREE PROBLEM WITH GIVEN DIAMETER IN A COMPLETE UNDIRECTED GRAPH ON UNI(0; 1)-ENTRIES

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We consider two well-known optimization problems: the Minimum Spanning Tree Problem and the Maximum Spanning Tree Problem. There are some extensions of these problems, for example, if we want to find extremal spanning tree with bounded maximum degree of vertices or we search for extremal spanning tree with bounded diameter from above or from below by some integer. The diameter of a tree is the number of edges in the longest simple path within the tree connecting a pair of vertices. In current work, we consider the intractable problem of finding a maximum-weight spanning tree with a given diameter in a complete undirected graph. We construct $O(n^2)$-time approximation algorithm solving the Maximum Spanning Tree Problem with a given diameter in a complete undirected n-vertex graph, and prove the sufficient conditions of asymptotic optimality for this algorithm in the case of independent uniformly distributed UNI(0; 1)-entries. This algorithm uses the algorithm for the Minimum Spanning Tree Problem with given diameter in a complete undirected graph.

Key words: Maximum Spanning Tree, Minimum Spanning Tree, approximation algorithm, probabilistic analysis, asymptotic optimality.

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АСИМПТОТИЧЕСКИ ТОЧНЫЙ ПОДХОД К РЕШЕНИЮ ЗАДАЧИ МАКСИМАЛЬНОГО ОСТОВНОГО ДЕРЕВА С ФИКСИРОВАННЫМ ДИАМЕТРОМ В ПОЛНОМ НЕОРИЕНТИРОВАННОМ ГРАФЕ С ВХОДНЫМИ ДАННЫМИ ИЗ КЛАССА UNI(0; 1)

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Мы рассматриваем труднорешаемую задачу отыскания максимального остовного дерева с фиксированным диаметром в полном неориентированном графе. Для приближенного алгоритма с трудоемкостью $O(n^2)$, решающего эту задачу, где n — количество вершин в графе, мы доказываем достаточные условия асимптотической точности в случае весов ребер графа из класса независимых, равномерно распределенных случайных величин UNI(0; 1).

Ключевые слова: максимальное остовное дерево, минимальное остовное дерево, приближенный алгоритм, вероятностный анализ, асимптотическая точность.

Introduction. One of the well-known discrete optimization problems is the Minimum Spanning Tree Problem. It consists of finding a spanning tree (connected acyclic subgraph, which covers all the vertices) of a minimal weight in a given undirected edge-weighted graph $G = (V,E)$. The polynomial solvability of the problem was shown in the classic algorithms by Boruvka (1926), Kruskal (1956) and Prim (1957). These algorithms have complexities $O(m \log n)$, $O(m \log m)$ and $O(n^2)$, respectively, where $m = |E|$ and $n = |V|$. Interested reader may refer to [1–2].

One of the possible generalizations of the above problem may be the bounded diameter Minimum Spanning Tree Problem. The diameter of a tree is the number of edges in the longest simple path within the tree connecting a pair of vertices. This problem is NP-hard in general for the diameter bounded above or from below to given number d.

We assume that weights of graph edges are independent identically distributed random variables from the class UNI(0; 1) of uniform distribution.

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Earlier, we have studied the bounded from above to given number d, or from below Minimum Spanning Tree Problem in directed and undirected graphs on UNI$(a_n; b_n)$-entries [3–9]. In the case of $a_n > 0$ we have implemented an asymptotically optimal approach for these problems.

In current paper we consider the Maximum Spanning Tree Problem with given diameter D in a complete Undirected graph (Given-D-Max-USTP). An ideologically close formulation of the problem was considered in the works [10–11], where an upper constraint was imposed on the radius R of the graph. In these works approximation $\mathcal{O}(n^2)$-time algorithms were presented and some performance ratios obtained on deterministic entries. The best of them, namely, $\mathcal{O}(\frac{1}{R})$, contains in [11].

In current work we describe an $\mathcal{O}(n^2)$-time approximation algorithm to solve the Given-D-Max-USTP and provide conditions for this algorithm to be asymptotically optimal in the case of UNI$(0; 1)$-entries.

Note, that proposed algorithm can be transformed to solve the Maximum Spanning Tree Problem with bounded diameter from below or above.

Statement 1. Let $w' : E \to (0; 1)$, $w : E \to (0; 1)$ be two weight functions such that $w'_e = 1 - w_e$. Let T be a tree with total weight $W'(T) = \sum_{e \in T} w'_e$, which is constructed in a graph $G = (V,E)$, $|V| = n$ with weight function (w'_e). Then T is a tree with total weight $W(T) = n - 1 - W'(T)$, which is constructed in a graph G with weight function (w_e).

The idea of finding an approximation Algorithm $\mathcal{A}$ for the Given-D-Max-USTP with weight function (w_e) is to construct an approximation Algorithm $\mathcal{A}'$ for the Minimum Spanning Tree Problem with given diameter in a complete Undirected graph (Given-D-Min-USTP) with weight function (w'_e) which according to Statement 1 the required solution for the problem.

1. The Algorithm $\mathcal{A}$ for the Given-D-Max-USTP. First of all, we introduce the Given-D-Min-USTP and approximation Algorithm $\mathcal{A}'$ for its solution. After that we describe the Given-D-Max-USTP and approximation Algorithm $\mathcal{A}$, which solves the Given-D-Max-USTP.

1.1. *The Given-D-Min-USTP.* Let $G = (V,E)$ be a complete edge-weighted undirected n-vertex graph with weight function (w'_e), the Given-D-Min-USTP is to find a spanning tree T with a given diameter $D = D_n < n$ of minimum total weight. We assume that the weights of the edges are independent and identically distributed random reals, with probability distribution function $f(x)$ defined on $(0; 1)$.

1.2. *The description of Algorithm $\mathcal{A}'$.* **Preliminary Step 0'.** In graph G choose an arbitrary $(D + 1)$-vertex subset V'.

Step 1'. Starting at arbitrary vertex of the subgraph $G(V')$ construct in it a Hamiltonian path P which consists of $D = D_n$ edges using the approach “Go to the nearest unvisited vertex”.

Step 2'. Each vertex of the subgraph $G(V \setminus V')$ connect by the shortest edge to the inner vertex of the path P. Thus, we add this edge to T.

The description of Algorithm $\mathcal{A}'$ is completed.

1.3. *The Given-D-Max-USTP.* Let $G = (V,E)$ be a complete edge-weighted undirected n-vertex graph with weight function (w_e), the Given-D-Max-USTP is to find a spanning tree T with a given diameter $D = D_n < n$ of maximum total weight. We assume that the weights of the edges are independent and identically distributed random reals, with probability distribution function $f(x)$ defined on $(0; 1)$.

1.4. *The description of Algorithm $\mathcal{A}$.* **Step 1.** Introduce weight function (w'_e) of graph G instead of initial weight function (w_e) such that $w'_e = 1 - w_e$.

Step 2. Apply Algorithm $\mathcal{A}'$ to the graph G with weight function (w'_e), obtain tree T.

Step 3. The constructed tree T is the solution for Given-D-Max-USTP.

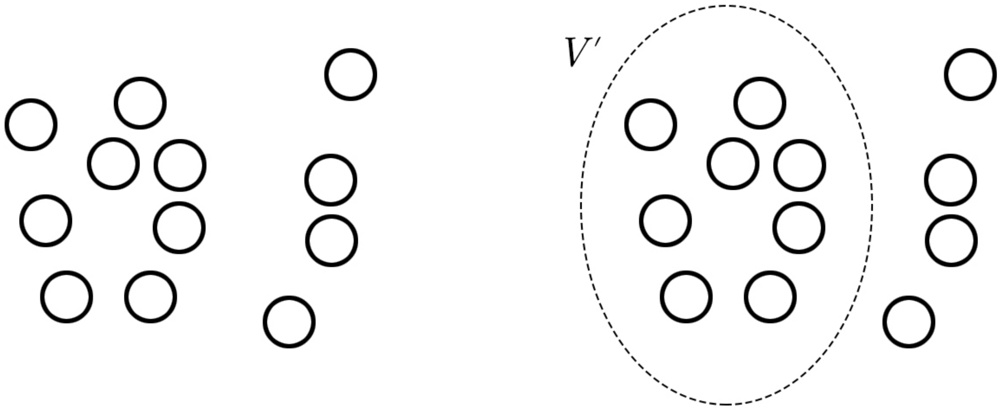

Fig. 1. Initial 12-vertex complete graph G and Step 0' of Algorithm $\mathcal{A}'$ in this graph, $D = 7$

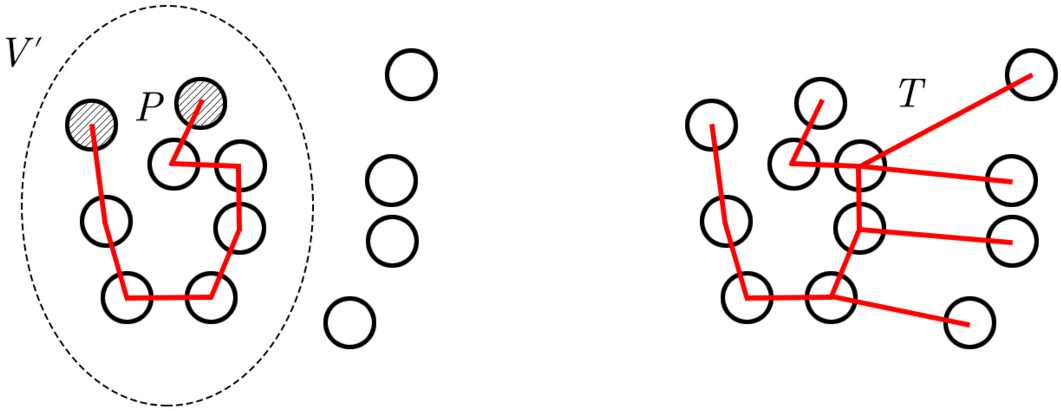

Fig. 2. Step 1' and Step 2' of Algorithm $\mathcal{A}'$ in 12-vertex complete graph G, $D = 7$. The hatched vertices are end vertices of path P

The description of Algorithm $\mathcal{A}$ is completed.

Denote by $W'_{\mathcal{A}'}$ the total weight of tree T constructed by Algorithm $\mathcal{A}'$. Let us formulate statements concerning Algorithm $\mathcal{A}'$ and Algorithm $\mathcal{A}$.

Statement 2. Algorithm $\mathcal{A}'$ constructs a feasible solution for the Given-D-Min-USTP.

Proof. A subgraph T consists of n vertices and $n-1$ edges since we firstly create the tree as the path on $D+1$ vertices during Step 1' and then we add edges to the tree by connecting all other vertices to the vertices in path on Step 2', finally, we obtain such construction, and we indeed get feasible solution for the Given-D-Min-USTP.

Statement 3. Algorithm $\mathcal{A}$ constructs a feasible solution for the Given-D-Max-USTP.

Proof. According to Statement 2 Algorithm $\mathcal{A}'$ finds feasible solution for the Given-D-Min-USTP and according to Statement 1 this solution with changed weight function is feasible solution for the Given-D-Max-USTP. Step 3 of Algorithm $\mathcal{A}$ doesn't change the constructed solution.

Statement 4. Time complexities of Algorithms $\mathcal{A}'$ and $\mathcal{A}$ are equal to $\mathcal{O}(n^2)$.

Proof. Preliminary Step 0' of Algorithm $\mathcal{A}'$ takes $\mathcal{O}(n)$ time.

At Step 1', the path P is built in $\mathcal{O}(D^2)$ time.

Step 2' takes $\mathcal{O}(n^2)$ operations since we connect all of $n-D-1$ vertices by the shortest edge to the inner vertex of the path P for the spanning tree T.

Thus, the total time complexity of the Algorithm $\mathcal{A}'$ is $\mathcal{O}(n^2)$. So Step 2 of Algorithm $\mathcal{A}$ requires $\mathcal{O}(n^2)$ operations.

Step 1 of Algorithm $\mathcal{A}$ is completed in $\mathcal{O}(n)$ actions. Step 3 of Algorithm $\mathcal{A}$ is carried out in time $\mathcal{O}(1)$.

Finally, the total running time of the Algorithm $\mathcal{A}$ is $\mathcal{O}(n^2)$ too.

2. A probabilistic analysis of Algorithm $\mathcal{A}$ for the Given-D-Max-USTP.

We perform the probabilistic analysis under conditions that weights of graph edges are random variables ξ from the class UNI$(0;1)$.

By $F_A(I)$ and $OPT(I)$ we denote respectively the approximate (obtained by some approximation algorithm A) and the optimum value of the objective function for the problem on the input I. An algorithm A is said to have *estimates (performance guarantees)* $\big(\varepsilon_n,\delta_n\big)$ on the set of random inputs of the n-sized problem (it is the problem with parameter n, where n is amount of input data required to describe the problem, see [12]), if

$$\mathbf{P}\Big\{|OPT(I)-F_A(I)|>\varepsilon_n OPT(I)\Big\}\leqslant\delta_n, \tag{1}$$

where $\varepsilon_n=\varepsilon_A(n)$ is an estimation of *the relative error* for the solution obtained by algorithm A, $\delta_n=\delta_A(n)$ is an estimation of *the failure probability* for the algorithm, which is equal to the proportion of cases when the algorithm does not hold the relative error ε_n or does not produce any answer at all.

Following [13] we say that an approximation algorithm A is called *asymptotically optimal* on the class of input data for the problem, if there exist such performance guarantees that for all input I of size n

$$\varepsilon_n\to 0 \text{ and } \delta_n\to 0 \text{ as } n\to\infty.$$

We denote random variable equal to minimum over k variables from the class UNI$(0;1)$ by ξ_k.

According to the description of Algorithm $\mathcal{A}'$ the weight $W'_{\mathcal{A}'}$ of the constructed minimum spanning tree T is a random value equal to W'. The weight of desired maximum spanning tree T equals $W=(n-1)-W'$. Let W'_1 and W'_2 be random variables for weight of edges added to T on Step 1' and Step 2' of Algorithm $\mathcal{A}'$, respectively. Obviously, $W'=W'_1+W'_2$.

Let ξ_k be random variable modeling the construction of the edge during the process of Algorithm $\mathcal{A}'$.

$W'_1=\sum_{k=1}^{D}\xi_k$ since we construct the path P consists of D edges during Step 1'.

$W'_2=(n-D-1)\xi_{(D-1)}$ since we connect each vertex from $G(V\setminus V')$, $|V\setminus V'|=n-D-1$ by the shortest edge to the $(D-1)$ inner vertex of the path P.

Lemma 1. For $\mathbf{E}(W')$ the next inequality is true

$$\mathbf{E}(W')\leqslant 2\ln n+\frac{n-1}{D}=\widehat{\mathbf{E}W'}.$$

Proof. Consider separately expectations of random variables for values W'_1 and W'_2.

$$\mathbf{E}(W'_1)=\sum_{k=1}^{D}\mathbf{E}(\xi_k)=\sum_{k=1}^{D}\frac{1}{k+1}\leqslant\ln D,$$

$$\mathbf{E}(W_2') = (n-D-1)\mathbf{E}(\xi_{(D-1)}) = \frac{n-D-1}{D} \leqslant \frac{n-1}{D}.$$

From the previous equations we get

$$\mathbf{E}(W') = \mathbf{E}\Big(W_1' + W_2'\Big) \leqslant \ln D + \frac{n-1}{D} \leqslant 2\ln n + \frac{n-1}{D} = \widehat{\mathbf{E}(W')}.$$

Lemma 1 is proved.

Lemma 2. Algorithm $\mathcal{A}$ for the Given-D-Max-USTP with weights of edges from UNI(0; 1) has the following estimates of the relative error ε_n and the failure probability δ_n:

$$\varepsilon_n = (1+\lambda_n)\frac{\widehat{\mathbf{E}W'}}{n-1}, \tag{2}$$

$$\delta_n = \mathbf{P}\Big\{\widetilde{W'} > \lambda_n \widehat{\mathbf{E}W'}\Big\}, \tag{3}$$

where $\lambda_n > 0$, $\widetilde{W'} = W' - \mathbf{E}W'$, and $\widehat{\mathbf{E}W'}$ is some upper bound for expectation $\mathbf{E}W'$.

Proof. Taking into account that in the case of the Given-D-Max-USTP the inequality $OPT \leqslant n-1$ is true, we have

$$\mathbf{P}\Big\{W < (1-\varepsilon_n)OPT\Big\} \leqslant \mathbf{P}\Big\{W < (1-\varepsilon_n)(n-1)\Big\} = \mathbf{P}\Big\{(n-1) - W' < (1-\varepsilon_n)(n-1)\Big\} =$$

$$\mathbf{P}\Big\{W' > \varepsilon_n(n-1)\Big\} = \mathbf{P}\Big\{W' - \mathbf{E}W' > \varepsilon_n(n-1) - \mathbf{E}W'\Big\} \leqslant \mathbf{P}\Big\{\widetilde{W'} > (1+\lambda_n)\widehat{\mathbf{E}W'} - \widehat{\mathbf{E}W'}\Big\} =$$

$$= \mathbf{P}\Big\{\widetilde{W'} > \lambda_n \widehat{\mathbf{E}W'}\Big\} = \delta_n.$$

Lemma 2 is completely proved.

Further for the probabilistic analysis we use the following probabilistic statement.

Theorem 1. (Petrov's Theorem [14]) Consider independent random variables $X_1, \dots, X_n$. Let there be positive constants Z and $h_1, \dots, h_n$ such that for all $k = 1, \dots, n$ and $0 \leqslant t \leqslant Z$ the following inequalities hold

$$\mathbf{E}e^{tX_k} \leqslant e^{\frac{h_k t^2}{2}}. \tag{4}$$

Set $S = \sum_{k=1}^n X_k$ and $H = \sum_{k=1}^n h_k$. Then

$$\mathbf{P}\{S > x\} \leq \begin{cases} \exp\{-\frac{x^2}{2H}\}, & \text{if } \ 0 \leq x \leq HZ, \\ \exp\{-\frac{Zx}{2}\}, & \text{if } \ x \geq HZ. \end{cases}$$

Lemma 3. Let ξ_k be random variable equal to minimum over k independent random variables from the class UNI(0; 1). Given constants $Z = 1$ and $h_k = \frac{1}{(k+1)^2}$. Then for variables $\widetilde{\xi_k} = \xi_k - \mathbf{E}\xi_k$ the condition (4) of Petrov's Theorem holds for each $t \leqslant Z$ and $1 \leqslant k < n$.

Proof. Evidently, $\mathbf{E}\xi_k = \frac{1}{k+1}$, denote $\alpha = \frac{t}{k+1}$. Using the formula

$$\mathbf{E}e^{t\xi_k} = \sum_{i=0}^{\infty} \frac{t^i}{(k+1)\cdots(k+i)}$$

(see in the book [15], p. 120), we estimate the value $\mathbf{E}e^{t\xi_k}$ from above:

$$\mathbf{E}e^{t\xi_k} \leqslant 1+\alpha+\alpha^2\frac{(k+1)}{(k+2)}\sum_{i=0}^{\infty}\Big(\frac{t}{k+3}\Big)^i = 1+\alpha+\alpha^2\cdot Q_{k,t} \leqslant e^{\alpha+\frac{\alpha^2}{2}} = e^{t\mathbf{E}\xi_k}\cdot e^{\frac{h_k t^2}{2}},$$

where $Q_{k,t} = \frac{(k+1)}{(k+2)(1-\frac{t}{k+3})} \leqslant Q_{k,Z} = \frac{(k+1)(k+3)}{(k+2)^2} < 1$ if $k \geqslant 1$. From this we have

$$\mathbf{E}e^{t(\xi_k-\mathbf{E}\xi_k)} = \mathbf{E}e^{t\widetilde{\xi}_k} \leqslant e^{\frac{h_k t^2}{2}}.$$

Lemma 3 is proved.

Lemma 4. The following upper bound for the sum of constants $h_k = \frac{1}{(k+1)^2}$ that correspond to edges of the spanning tree T is true.

$$H \leqslant \psi + \frac{n}{D^2},$$

where $\psi = 0.645$.

Proof. The parameter H equal to the sum of H_1 and H_2 according to the steps of Algorithm $\mathcal{A}'$ number 1' and 2', respectively. Knowing that notation and estimates from above, we obtain

$$H_1 = \sum_{k=1}^{D} h_k = \sum_{k=1}^{D}\frac{1}{(k+1)^2} < \psi.$$

We have used the well-known Euler's estimate for the inverse square equation $1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots = \frac{\pi^2}{6} < 1.645$ in the calculation of H_1.

For parameter H_2, we get

$$H_2 = \big(n-(D+1)\big)h_{(D-1)} \leqslant \frac{n}{D^2}.$$

Finally, we obtain

$$H = H_1 + H_2 < \psi + \frac{n}{D^2}.$$

Lemma 4 is completely proved.

Theorem 2. The Given-D-Max-USTP with weights of edges from UNI$(0;1)$ can be solved by Algorithm $\mathcal{A}$ with the relative error

$$\varepsilon_n = \mathcal{O}\Big(\frac{2\ln n}{n} + \frac{1}{D}\Big) \tag{5}$$

and the failure probability

$$\delta_n = n^{-1} \to 0, \text{ as } n \to \infty. \tag{6}$$

Proof. Let us remind that $W = (n-1) - W'$, so in Petrov's Theorem we interest in values, which are equal to sums of minimum ξ_k.

Set $\lambda_n = 1$. First of all, we prove the equation (6). From Lemma 2 and the upper bound for $\mathbf{E}W'$ (see Lemma 1) we have

$$\varepsilon_n = (1+\lambda_n)\frac{\widehat{\mathbf{E}W'}}{n-1} = \frac{2}{n-1}\Big(2\ln n + \frac{n-1}{D}\Big) = \mathcal{O}\Big(\frac{2\ln n}{n} + \frac{1}{D}\Big).$$

Note that in the course of the Algorithm $\mathcal{A}'$ we have deal with random variables of the type ξ_k, $1 \leqslant k \leqslant D$. In the case of graphs with weights of edges from UNI$(0;1)$, these variables satisfy the conditions of the Petrov's Theorem for constants $Z = 1$ and $h_k = \frac{1}{(k+1)^2}$ (see Lemma 3). Now using Petrov's Theorem and Lemma 2, we estimate the failure probability:

$$\delta_n = \mathbf{P}\big\{\widetilde{W'} > \lambda_n \widehat{\mathbf{E}W'}\big\} = \mathbf{P}\Big\{\widetilde{W'} > 2\ln n + \frac{n-1}{D}\Big\}.$$

Define constants $Z = 1$ and $h_k = \frac{1}{(k+1)^2}$ for each variable whose weight corresponds to a random variable ξ_k and which are included to the constructed spanning tree.

From Lemma 4 we have

$$ZH \leqslant \psi + \frac{n}{D^2} < 2\ln n + \frac{n-1}{D} = x.$$

According to Petrov's Theorem, we have an estimate for the failure probability of Algorithm $\mathcal{A}'$:

$$\delta_n = \mathbf{P}\{\widetilde{W'} > x\} \leqslant \exp\Big\{-\frac{Zx}{2}\Big\}.$$

Given the value x we have

$$\frac{Zx}{2} = \frac{1}{2}\Big(2\ln n + \frac{n-1}{D}\Big) \geqslant \ln n.$$

From this, it follows that

$$\delta_n = \mathbf{P}\{\widetilde{W'} > x\} \leqslant \exp\Big\{-\frac{Zx}{2}\Big\} < \exp\{-\ln n\} = \frac{1}{n} \to 0.$$

Theorem 2 is completely proved.

We have the evident corollaries from this Theorem.

Corollary 1. The Given-D-Max-USTP is solved asymptotically optimal in the case of $D = D_n \to \infty$, for example, for $D \geqslant \ln n$.

Corollary 2. In the case of the Given-D-Max-USTP on entries from UNI$(a_n; b_n)$, $a_n \geqslant 0$, Algorithm $\mathcal{A}$ has the same conditions of asymptotic optimality as for UNI$(0;1)$-entries without additional condition for ratio b_n/a_n, which appears in minimum problems on UNI$(a_n; b_n)$-entries, $a_n > 0$ (see [3–9]).

Conclusion. In this work, we have described an $\mathcal{O}(n^2)$-time approximation algorithm for the Maximum Spanning Tree Problem with given diameter in a complete edge-weighted undirected graph. In the case of uniform distribution UNI$(0;1)$ for the weights of the graph edges, we have obtained estimates of performance guarantees for solving the problem and suggested sufficient asymptotic optimality conditions for obtained solution. It would be interesting to investigate this problem on input data with infinite support like exponential or truncated-normal distribution and on discrete distributions. It is interesting to consider asymptotic optimality for the problem of finding several edge-disjoined Maximum Spanning Trees with a given or bounded diameter.

Эдуард Хайрутдинович Гимади — тел. 329-75-41, e-mail: gimadi@math.nsc.ru. Доктор физико-математических наук, профессор, главный научный сотрудник Института математики им. Соболева СО РАН — специалист в области математической кибернетики, дискретной оптимизации и исследования операций. В Институте математики Э. Х. Гимади работает с мая 1965 года. Основным направлением научной деятельности Э. Х. Гимади является разработка эффективных приближенных алгоритмов решения трудных задач дискретной оптимизации в исследовании операций и обоснование оценок

их качества. Особое место в этом направлении занимает асимптотически точный подход к решению задач дискретной оптимизации.

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